Choose a game that you are most interested in and try to invent a strategy for one of the players (the first or the second) that guarantees them a win regardless of the opponent's moves. Try not only to describe the strategy but also to explain why the win in this case is inevitable. An answer without any explanation cannot be accepted.
Do not try to solve all the problems, save time and energy for other competitions. A good analysis of even one game allows to consider your participation in the competition as successful.

Task 1. "Two snakes".
Two people take turns in covering the squares of a field $m \times n$ with paint, each with their own colour. As a first move, they paint over the opposite corner squares. Next, each one goes on with their own "snake" every time painting over a square adjacent to the side of the one the they have painted over earlier. The rival "snakes" are prohibited from touching any side of the rival squares. The one who cannot make a move, loses the game. Who — the one who starts or their opponent — wins this game regardless how well their partner plays? Consider the following cases:

a) $m = n = 9$;
b) $m = 8$, $n = 10$;
c) $m = 9$, $n = 10$;
d) $m = 2$, $n = 15$.

Task 2. "Coins".
There are N coins in a jar (N is an odd number). Two players take turns in taking the coins out of the jar. During a move, one can take one or two coins. The game is over when the jar is empty. Who wins having a right strategy, the one who starts or their opponent if:

a) $N = 7$, and the one who has an **even** number of coins after the game is over, wins;
b) $N = 7$, and the one who has an **odd** number of coins after the game is over, wins;
c) $N = 9$, and the one who has an **even** number of coins after the game is over, wins?

d) Consider the general case: who wins depending on N if winning requires an even number of coins in the end, and who wins if winning reuires an odd number of coins in the end?

Task 3. "Scales".
There are N weights that weigh 1, 2, 3,. . . , N grams. Two players take turns in putting the weights on the scale. Each player puts the weight on their weighing pan so that their weighing pan will weigh down the scale after their move. The one who cannot make a move according to the rules is considered to be *the winner*.
Who wins using a right strategy, the one who starts or their opponent if:

a) $N = 4$;
b) $N = 7$;
c) $N = 8$;
d) $N = 99$;
e) $N = 98$?

f) Petya and Vasya played for $N = 8$, Petya started. As a result, all eight weights were put on the scale, and Petya won. Prove that Vasya could have won but missed his chance.

Don't forget to **sign** your work (please, write the card number, your last name, school and grade) before **submitting** the work. You do not have to submit the sheet with the tasks. The tasks, their solutions and the results of the competition will be published at *http://turlom.olimpiada.ru* after November 20.