XXXIX Lomonosov Tournament, September 25, 2016

Competition on mathematical games

Choose a game that you are most interested in and try to invent a strategy for one of the players (the first or the second) that guarantees them a win regardless of the opponent's moves. Try not only to describe the strategy but also to explain why the win in this case is inevitable. An answer without any explanation cannot be accepted.

Do not try to solve all the problems, save time and energy for other competitions. A good analysis of even one game allows to consider your participation in the competition as successful.

1. «Paper Clips» There is a chain of paper clips of several different colours. Two people play. In one turn a player may disengage any two paper clips of ***different*** colours and link the resulting parts in one chain, connecting the clips of the ***same*** colour. A player who cannot make a move loses the game.

Who — the one who starts or their opponent — will win the game regardless of the other player's strategy? (Obviously, the answer may depend on the original chain.)

Consider the following cases (for notational convenience let us denote the colours by numbers):

a) Chain **213312**;

b) Chain **121314151617181912**;

c) A chain with paper clips of two colours, the location of the clips is not specified

d) A chain with paper clips of three colours, the location of the clips is not specified;

e) Create a chain of six paper clips (choose the number of colours by yourselves) such that it allows the first player to win unless they make a wrong move: in this case, the second player wins. Show how the game will progress of in either case.

2. «Divisors» Initially the number **1** is written on the blackboard. Two people play the game. In one turn the player may add to the number written on the blackboard any of its divisors (including **11** and the number itself), then write the new number on the blackboard and erase the old one. You cannot write numbers greater than $\boldsymbol{N}$. The winner is the first player to write $\boldsymbol{N}$.

Who will win (providing a good strategy): the first player or their opponent?

Consider the following cases:

a) $\boldsymbol{N = 6}$;

b) $\boldsymbol{N}$ — is any odd number;

c) $\boldsymbol{N = 1024}$;

d) $\boldsymbol{N = 120}$;

e) $\boldsymbol{N = 2p}$, where $\boldsymbol{p}$ is a prime number.

f) You are playing the game until $N = 42$. Now the number "15" is written on the boad and it's your turn. Can you surely win?

3. «Far off the footworn roads» There is a chip in the corner square of a checkered board of size $\mathbf{m} \times \mathbf{n}$. Two players move the chip in turns. In one turn the player can move the chip to an adjacent square. You cannot move the chip to the squares where the chip has already been placed, and to the squares adjacent to those (the current square is not taken into account). The player who cannot make a move loses the game.

Who wins in case of having a good strategy: the one who makes the first move or their opponent?

Consider the following cases:

a) $\mathbf{m = n = 8}$;

b) $\mathbf{m = 4}$, $\mathbf{n = 5}$;

c) $\mathbf{m = 2}$, $\mathbf{n}$ is any number;

d) $\mathbf{m = 3}$, $\mathbf{n}$ is any number;

e) $\mathbf{m}$ and $\mathbf{n}$ are any odd numbers.

Do not forget to **sign** your work (Please write the card number, your last name, school and grade) and **hand the work in**. You do not have to hand in the sheet with the tasks. The tasks, the problem solutions and the results of the competition will be available at **turlom.olimpiada.ru** after 20 November.