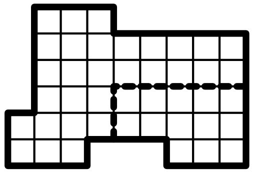
You need to choose one of the three games below (the one which is most interesting for you) and to come up with a winning strategy for the 1st or the 2nd player. Your strategy must guarantee a victory regardless of the opponent's moves. Try not only to describe the player's moves but also to explain why the victory is inevitable. A solution without proper explanation does not count.

Do not rush into solving all the tasks as you need to save time and energy for other competitions. A good analysis of even a single game will be considered as a success.

1. "Corner sections". From a list of checkered paper, a figure is cut out along borders of cells. The first player cuts it along borders of cells into two parts such that the section is of the form of the greek letter " Γ ", that is, consists of two perpendicular segments.



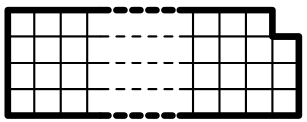
The second player proceeds in the same way with any one of two obtained figures, then the first one proceeds so with one of three obtained figures, and so on. A player who cannot make a move loses. Which of two players wins in this game independently of moves of the other one?

Consider the cases when the initial figure is

a) a square 3×3

b) a square 4×4

c) a rectangle $4 \times N$ with N an arbitrary positive integer, such that one corner cell is cut off



d) a rectangle 4×7

e) a square 5×5 such that one corner cell is cut off

f) a square $(N+4) \times (N+4)$ such that a corner square $N \times N$ is cut off, N an arbitrary positive integer

g) a rectangle 13×25

2. "A pack of cards". A pack of cards includes two red and many black cards. Two players always see the positions of cards in the pack.

Each player in turn takes any one of red cards and puts it in any place of the pack which is above its former position. If two red cards are neighbouring then the player may take both and put them in the pack together but also above their former position. One wins when both red cards are above the pack.

It is known that the upper of two red cards is

a) second from top

b) third from top

c) fourth from top

Which of the players wins according to a given position of the second red card?

d) Describe all possible cases of layout of red cards such that the second player wins.

3. "A hotel". In a hotel, two managers play the following game. There are N identical rooms in the hotel. Initially, there is a single guest in each room. Each manager in turn moves all guests from some room to some another room and starts repair in the first room. The number of guests in any room must not exceed the number of places in this room. A manager who cannot make a move loses.

Which player wins in this game independently of moves of the other one?

Consider the following cases:

a) each room is for 6 persons, $N = 10$

b) each room is for 3 persons, $N = 15$

c) each room is for 3 persons, $N = 17$

d) each room is for 3 persons, N arbitrary

e) each room is for 4 persons, N arbitrary