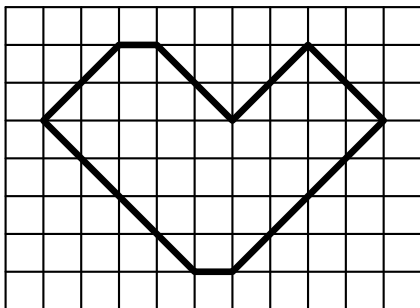


Mathematics Competition

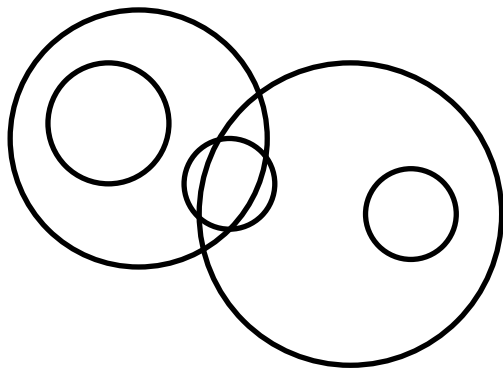
For each problem, the recommended grades are specified in the parentheses in front of the problem. A student is allowed to solve problems intended for older grades. The problems for younger grades do not count for the results. (The 11th grade in Russian school is the last year before graduation.)

1. (6–7) Bratsk, Gusev, Komsomolsk-on-Amur, Zlatoust, and Elizovo are five Russian cities. When it is noon in Bratsk, it is 6 AM in Gusev and it is 2 PM in Komsomolsk-on-Amur. When it is noon in Zlatoust, it is 6 PM in Elizovo, and it is 9 AM in Gusev. Can you tell what time it is in Komsomolsk-on-Amur when it is noon in Elizovo?

2. (6–7) Cut this shape into three identical pieces. (“Identical” means “having the same shape and the same size”.)

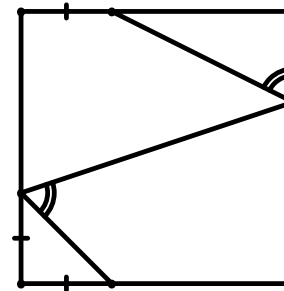


3. (6–8) A forest ranger was inspecting his forest. At the picture below, the five circles represent the five sectors that the ranger inspected. The ranger claims that exactly 3 pine trees grow in each sector. Is it possible that all the claims of the ranger are correct?



4. (8–9) Can you come up with a number that is divisible by exactly 50 numbers from this list: 1, 2, 3, . . . , 100?

5. (8–9) The four segments marked on the sides of the square are identical. (See the picture.) Prove that the two marked angles are of the same size.



6. (7–11) One hundred parrots were sting on a tree. Some of these parrots were green, some – yellow, and others – spotted. A crow landed on the tree.

“There are more green parrots than spotted parrots on this tree!” – the crow cowed. Exactly 50 parrots replied with “yes”, and the rest replied with “no”.

“There are more spotted parrots than yellow parrots on this tree!” – the crow cowed. Again, exactly 50 parrots replied with “yes”, and the rest replied with “no”.

It is known that during this conversation every green parrot told the truth both times, every yellow parrot lied both times, and every spotted parrot lied ones and told the truth once. Is it possible that there were more yellow parrots than green parrots on this tree?

7. (10–11) All terms of an infinite arithmetic progression are different natural numbers. For each term of this sequence, a square root was calculated and rounded to the nearest integer. Is it possible that all these numbers were rounded in the same way: either all were rounded up, or all were rounded down?

8. (10–11) Let us consider polyhedrons that possess the following property. For any two vertices of such polyhedron it is possible to find a third vertex such that these three vertices together form an equilateral triangle. The regular tetrahedron has this property. Are there any other polyhedrons like this?