

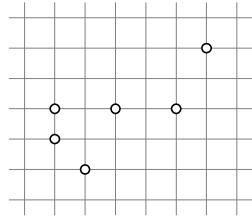
Numbers in brackets stand for the grades, for which the problems are recommended (you can also solve the problems for the higher grades, the problems for lower grades are not taken into account while calculating the results).

Task 1. (6–7) Four little mice, White, Grey, Thick and Thin shared a head of cheese. They made it into four seemingly equal parts. Some parts had more holes, than others, that's why the Thin's part was 20 grams lighter, than Thick's part, and White's part was 8 grams lighter, than Grey's part. But White didn't get upset, because his part weighted exactly a quarter of cheese weight.

Grey cut 8 grams of cheese off his part, Thick cut 20 grams off his part. How should the little mice divide these 28 grams of cheese if they want to all have equal parts? Don't forget to explain your decision.

Task 2. (6–8) There are six dots put on squared paper (see figure 1). Draw three straight lines so the following conditions would be satisfied:

- each dot lies on at least one of the lines
- on each line there are at least two dots
- three lines intersect at a single point (with or without a dot).



Task 3. (6–8) Ilya has a 3×3 table filled with numbers from 1 to 9 (see figure, on the left). In one move he can interchange either any two columns or any two rows. Can he come to the table on the right in a few moves?

1	2	3
4	5	6
7	8	9

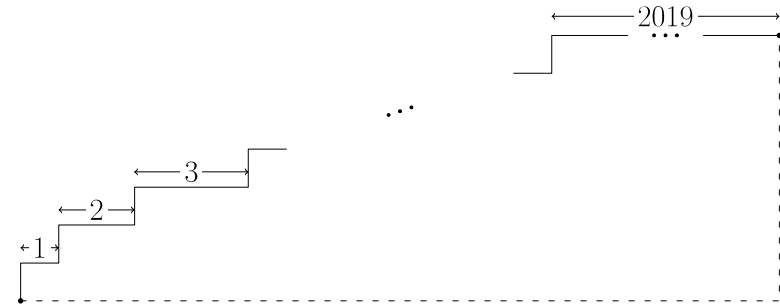
1	4	7
2	5	8
3	6	9

Task 4. (8–9) Let a, b, c, d and n be positive integers. Prove that if $(a - b)(c - d)$ and $(a - c)(b - d)$ are divisible by n , then $(a - d)(b - c)$ is also divisible by n .

Task 5. (9–11) There was a table tennis tournament held in a school. In each round each of the participants played exactly one game, and a referee of each match was one of the participants who was not playing.

After several rounds each participant played exactly one time versus each of the other participants. Is it possible that every participant was a referee of the same number of games?

Task 6. (9–11) The height of each step of the “stairs” (see figure) is 1, and the width of each step increases from one to 2019. Is it true that the segment from the lower left point of the stairs to the upper right point of the stairs doesn't cross the stairs?



Task 7. (10–11) The sum of some positive numbers equals one. Prove that among them there is a number which is not less than the sum of squares of all the numbers.

Don't forget to **sign** your work (please, write the card number, your last name, school and grade) before **submitting** the work. You do not have to submit the sheet with the tasks. The tasks, their solutions and the results of the competition will be published at <http://turlom.olimpiada.ru> after November 20.